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Two-dimensional axisymmetric solitons in nematic liquid crystals

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Two-dimensional axisymmetric propagating single solitons are generated experimentally in a radial Poiseuille flow of homeotropic nematic liquid crystals. In different pressure gradient regimes two types of single solitons are excited and identified. Interference patterns of transmitted light through crossed polarizers are photographed. Distributions of the director corresponding to these solitons are discussed.

1. Introduction

One-dimensional solitons are now becoming, relatively speaking, understood and have been found to be important in various physical phenomena [1, 2]. In contrast, investigation of solitons in higher dimensional space [2] is still in its infancy. Nevertheless, since many real physical systems are not strictly one-dimensional, the study of these solitons is urgent and imperative. In the case of two-dimensional solitons several theoretical studies have been made [3], but surprisingly few experimental observations have been carried out [4–6] to our knowledge [7]. The word 'soliton' used in this paper is synonymous to 'solitary wave', conforming to the common practice in the condensed matter [2] and particle physics literature [8].

In this paper we report on experiments on two-dimensional axisymmetric single solitons with constant velocities and widths in nematic liquid crystals. They are generated in homeotropic nematic disc cells in which pressure is applied at the rim, and which can even be observed by the naked eye. In contrast, the two-dimensional 'walls' observed by Leger [5] are elliptic in shape and much smaller in size and must be observed by microscopy. More importantly, the creation of these elliptic walls cannot be controlled at will. One has to search for them in the sample if and when they exist.

Solitons in liquid crystals [9] are important partly because they can be controlled and optically observed with ease, and partly because they are related to switching processes in display devices [9, 10]. They are found also in convective patterns [11] on the route to chaos. In uniform shearing nematics propagating one-dimensional solitons were first discussed by Lin *et al.* [12]. Evidence of these solitons is found [12] in the non-linear wave experiments of *N*-(*p*-methocybenzylidene)-*p*-butylaniline (MBBA) [13]. In [13] the solitons are excited by a pushing plate and multisolitons [9, 14] (corresponding to three dark lines under transmitted white light) are in fact observed. Moreover, near the pushing plate the flow of nematic is very complicated and the mechanism of soliton generation in this type of experiment is not at all clear. In this regard, the experiments described in this paper constitute a new way of generating solitons in shear-related nematics, the mechanism of which is relatively simple.

2. Experimental

MBBA is placed in a homeotropic disc cell of radius 5 cm and thickness 20 μm. The upper glass plate has a small hole in the centre and four identical holes near the rim. The latter are situated symmetrically and connected by a circular groove cut on the inner side of the upper plate. The radius of each hole is 0·5 mm. The pressure at the centre (p_c) and at the rim (p_r) are controlled independently. White light is transmitted perpendicularly to the cell through crossed-polarizers between which the cell is sandwiched. Interference patterns can be observed by the eye and are photographed at regular time intervals of Δt. The cell is maintained at constant temperature of 25 ± 0·5°C. Details of the cells and experimental set-up have been reported elsewhere [15] and will not be repeated here.

Throughout this experiment, $p_c = p_0 = 1$ atm. p_r changes with time according to figure 1, where $0{\cdot}07\,\mathrm{cm\,Hg} \leqslant p \leqslant 30\,\mathrm{cm\,Hg}$, $t_2 - t_1$ and $t_3 - t_2$ are both equal to 5 s. For convenience the region $t_1 \leqslant t < t_2$ ($t_2 \leqslant t < t_3$) is denoted as stage I (II). In stage I, nematic flows towards the centre; in stage II, it flows outward. Experiments are performed for three different regimes of p.

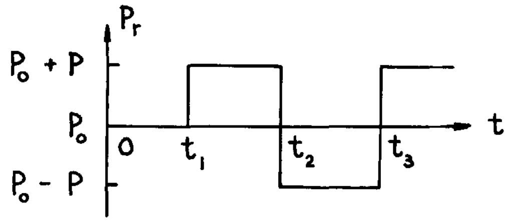
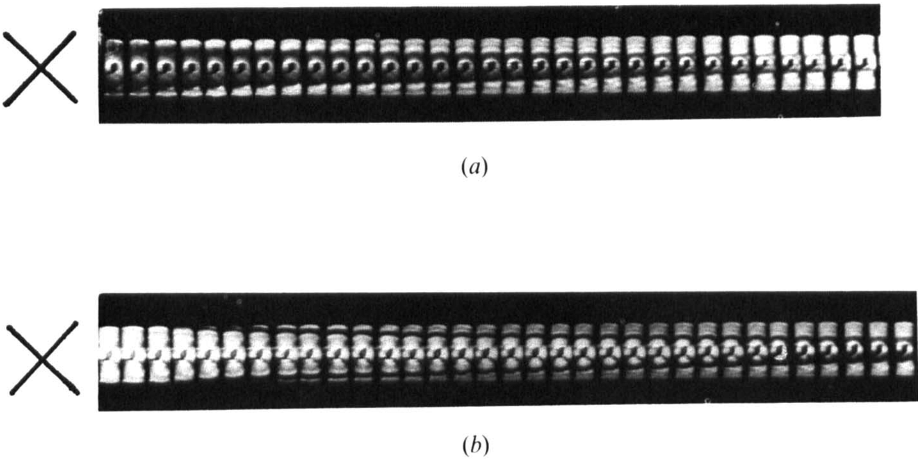


Figure 1. Variation of rim pressure p_r with time t.

3. Results and discussion

Low p regime. $p < p_1$ ($= 2{\cdot}5$ cm Hg). In stage I the appearance of the cell changes from all dark (corresponding to the director **n** being vertical) to white starting from the rim as shown in figure 2 (*a*), in which only a strip of the disc is shown from lack of space in the figure. The dark area shrinks towards the centre until the whole disc becomes white. In stage II a dark ring appears at the rim and moves towards the centre with constant velocity (6·9 cm/s) and width (6·6 mm) until it vanishes at the centre (figure 2 (*b*)).

These results are similar to those obtained in long liquid crystal cells [16] and may be understood as follows. At the end of stage I the flow is steady and radially inward. **n** is in the ($\hat{r}$, $\hat{z}$) plane and forms a steady distribution, where r is the radial coordinate and $\hat{z}$ is the normal of the cell. As in the uniform shear case **n**, in the central layer of



(*a*)

(*b*)

Figure 2. Interference patterns in low p regime ($p < p_1$). $p = 2{\cdot}0\,\mathrm{cm\,Hg}$, $\Delta t = 0{\cdot}0155\,\mathrm{s}$. The small central white circle is due to the light source. The short dark line at the centre is the shadow of the connecting tube. Here and in figures 3 and 5 the time axis is towards the right; the cross represents the two crossed directions of the two polarizers. (*a*) Stage I; (*b*) stage II.

the upper and lower halves of the cell [12] at which the shear is large, is inclined towards the centre. In the beginning of stage II the pressure gradient reverses sign. The previous distribution of molecular velocity, $\mathbf{v}$, and $\mathbf{n}$ is no longer the steady state. Near the rim where the pressure is changed the molecules are in the new steady state (with $\mathbf{v}$ outward and $\mathbf{n}$ inclined away from the centre) while near the centre they are still in the old steady state. The molecules in the transition region are vertical and appear as dark lines under white light. As the new steady state is favoured the corresponding region expands and the transition region (the dark line) thus moves toward centre. The set of faint dark rings appearing in figure 2 (and also in figure 3), in addition to the major dark ring, represent the intensity variation of transmitted white light and not vertical molecules. This has been confirmed by measuring the transmitted light intensity at fixed points in space.

Intermediate p regime. $p_1 < p < p_2$ ($= 4{\cdot}0\,\mathrm{cm\,Hg}$). At the beginning of stage I there are two white regions separated by a dark region. The former are situated near the rim and at the centre, respectively. (Note that the white region at the centre is absent in the low p regime.) As time passes the dark region shrinks gradually to a certain minimum width at about half the radius of the disc. It then moves towards the centre and there vanishes. The propagating dark region appears as a dark ring with constant velocity (12·1 cm/s) and width (1·4 mm) as shown in figure 3. In contrast, the dark ring in figure 2(*b*) appears in stage II (not I) and starts from the rim.

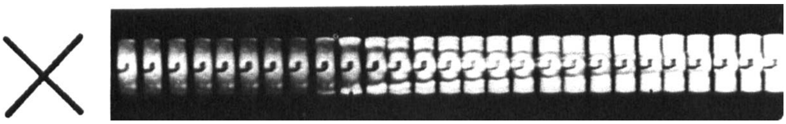


Figure 3. Interference patterns in stage I of the intermediate and high p regimes ($p > p_1$). $p = 3{\cdot}0\,\mathrm{cm\,Hg}$, $\Delta t = 0{\cdot}0155\,\mathrm{s}$.

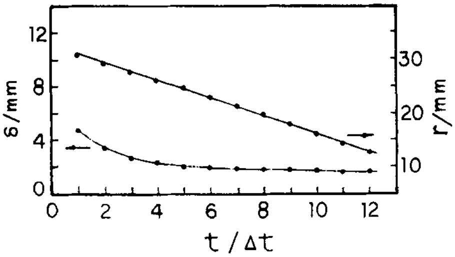


Figure 4. Variation of the location of the centre (r) and width (δ) of the dark region (which evolves into a dark ring at $r \sim 2{\cdot}5$ cm) as functions of time t. The two lines are drawn to guide the eye. The slope of the r versus t straight line gives a soliton velocity $C = 10{\cdot}9$ cm/s, $\Delta t = 0{\cdot}015$ s, temperature is 32·6°C.

In stage II, where $p_1 < p < p_3$ ($\gtrsim p_1$) the same phenomena as in the case of $p < p_1$ are observed. When $p_3 < p < p_2$, gray and white rings appear in the dark-brush regions corresponding to the directions of the two polarizers. The number of these rings varies with p. A complicated group of dark and white rings moving towards the centre (not shown here) appears on the whole disc.

High p regime. $p > p_2$. The result of stage I is identical to that of the intermediate p regime. A typical result of the variation of the location and width of the dark region and eventually the dark ring is shown in figure 4. The steady values of the velocity and width of the dark ring are essentially independent of the pressure applied [17]. In stage II (figure 5), a white ring is formed at the rim and moves with constant velocity ($\sim 17{\cdot}1$ cm/s) and width ($\sim 3{\cdot}7$ mm) towards the centre. The brightness of this white ring is not diminished in the dark-brush region.

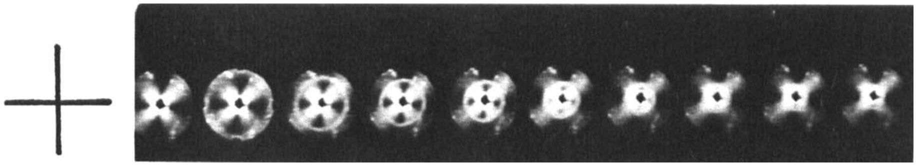


Figure 5. Interference patterns in stage II of high p regime ($p > p_2$). $p = 6{\cdot}0$ cm Hg, $\Delta t = 0{\cdot}033$ s. Starting from the left, the first pattern shows four dark brushes separated by white (or grey) regions. The second one shows the formation of a white ring at the rim, which has already entered the dark-brush regions. The rest shows the shrinking of the white ring towards the centre.

The above results of intermediate and high p regimes may be interpreted as follows. In stage I and $p > p_1$, instability (resulting in a deviation of $\mathbf{n}$ from the vertical direction) appears in regions not only near the rim but also at the centre. The former region corresponds to molecules in the new steady state. The latter corresponds to molecules in a perturbed state resulting from the jamming of molecules at the centre when all the molecules are rushing radially inward due to high pressure at the rim. This perturbed state is not necessarily the same as the steady state near the rim although both of them will appear white under white light. Consequently, when these two white regions meet each other, the dark region in between will not disappear but only move towards the centre.

In stage II and $p > p_3$, it is always observed that locally the dark brushes do turn white at some time. In our experimental set-up this indicates that, in the local region,

n is out of the $(\hat{r}, \hat{z})$ plane. (Through crossed polarizers, the transmitted light intensity $I \sim \sin^2 2\varphi$ if **n** is in the $(\hat{r}, \hat{z})$ plane, where φ is the angle between $\hat{r}$ and the first polarizing direction. In the four symmetrical directions of the two crossed-polarizers, $\varphi = 0°$ or $90°$ resulting in $I = 0$ and dark brushes. When **n** goes out of the $(\hat{r}, \hat{z})$ plane the simple relationship between I and φ here is no longer valid. It is then possible for the dark brushes to turn white locally.) Due to symmetry of the cell these local regions are always in a ring shape.

For $p_3 < p < p_2$, stage II is somewhere in between. When the pressure gradient is reversed in sign **n** enters the third dimension, i.e. out of the $(\hat{r}, \hat{z})$ plane. The propagation of this perturbed state results in the group of dark and white rings passing through the dark brushes as observed.

The above discussion for $p > 0$ is equally applicable for $p < 0$. In fact, experimentally we observed the same phenomena for both cases. When the thickness of the cell is varied similar results are observed except that the values of p_1, p_2 and p_3 also vary.

The observed results are reproducible when the pressure is cycled repeatedly, i.e. no hysteresis effects exist. When the negatives of the photographs are examined under a microscope no anisotropy of the rings is detected. Even though there are only four holes at the rim the groove connecting them effectively produces an isotropic pressure at the rim. The boundaries near the rim and the centre and the transient effects when the pressure is suddenly reversed will only affect the initial formation and final decay but not the existence of the solitons [9, 15]. The solitons are ring-shaped and called two-dimensional as is the custom in the soliton literature [3, 4]. Preliminary theoretical discussions can be found in [9] and [16]. We note that white rings have been recorded in [18] in which *steady* states of director under steady radial Poiseuille flow are studied. There is no mention of their propagating behaviour or conditions of generation.

In short, we have presented a new way of generating solitons, in this case *single* solitons, in nematic liquid crystals. They appear as dark or white rings moving towards the centre. In stage I, there are two possible cases: for $p < p_1$ there is no dark ring; for $p > p_1$ there is a dark ring starting from the middle of the radial length of the cell. In stage II, there are three possibilities: for $p < p_3$ there is a dark ring; for $p > p_2$, a white ring; for $p_3 < p < p_2$, a group of dark and white rings. The physical system here may be a good candidate for studying the problem of order and chaos [19].

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